The Sandreckoner’s Gifts

A Brief Profile of Archimedes of Syracuse

By Rafael Olivas

The year is 212 BC, turbulent times in the ancient world. The place is Syracuse, a modest seaport on the eastern coast of Sicily. Here was born a man, now about to die. His name is Archimedes, an aging eccentric Greek, well known for his engineering and mathematical genius. Yet, even at 75 he will die too young—and not peacefully. Fortunately for us, not all of his gifts will die with him.

Founded by Greek colonists several centuries earlier, Syracuse finds itself caught between rival powers as the Second Punic War ensues. Rome and Carthage wrestle each other for dominion of the Mediterranean world. In years past King Hiero II of Syracuse, considering the city’s vulnerability, commissioned defenses to ward off whichever power might attack. Archimedes turned his considerable talents to engineering a variety of ingenious and lethal devices. He may have done so with considerable ambivalence. Plutarch suggests that Archimedes thought that building such engines of war a vulgar thing; indeed, the entire enterprise of building objects for “mere use and profit.” [1]

No matter, in 210 BC, it is the Romans who attack the city by land and sea, only to find that one man’s cleverness will rob them of easy victory. Polybius reports that, “Archimedes had constructed artillery which could cover a whole variety of ranges… and so demoralized the Romans that their advance was brought to a standstill.”[2] He also contrived short-range defenses against soldiers who made it as far as the walls. Here Archimedes “had the walls pierced with large numbers of loopholes at the height of a man… Behind each of these and inside the walls were stationed archers with rows of so-called 'scorpions', a small catapult which discharged iron darts.”[3]

The Roman navy fares no better. The engineer had designed industrial-strength cranes to deter ships that might carry soldiers to the seaward walls. These machines use pulleys, chains and hooks to magnify physical labor. A small team of men operating the crane can grapple a ship with the giant hook, pluck it out of the water, and topple it back into the harbor as a wreck. Marcellus, the frustrated Roman general, is forced to offer grudging respect for his opponent: “Archimedes uses my ships to ladle seawater into his wine cups!”[4] And so the mighty Roman army and navy, defeated in direct assault, resort to siege in the shadow of an engineering genius.

But this is not the extent of the Archimedian gifts, nor the reason we should mourn his death or celebrate his life.

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[1] Plutarch, c. 46 – 119 CE, *Parallel Lives: Marcellus*. (C. Rorres) “…but, repudiating as sordid and ignoble the whole trade of engineering, and every sort of art that lends itself to mere use and profit, he placed his whole affection and ambition in those purer speculations where there can be no reference to the vulgar needs of life…”

[2] Polybius, c. 200-118 BC, *The Histories*, Book VIII , Translated by Ian Scott-Kilvert.

[3] Ibid.

[4] Ibid.

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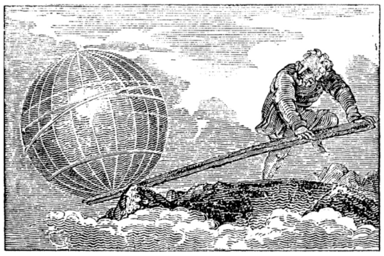
Give me a place to stand and I will move the Earth.[5]
—Archimedes, on explaining the potential of the lever and fulcrum.



Engraving from *Mechanics Magazine,* London, 1824

Archimedes was born around 287 BC, into a lively Hellenic culture. His father Phidias was an astronomer and must have encouraged his son towards intellectual pursuits. A reference in Archimedes' *The Sandreckoner* acknowledges his father's contributions. Evidence suggests that a young Archimedes probably studied at Alexandria, near present day Cairo. If so he would have been exposed to the brightest minds at the greatest research center of his day. The famous Library at Alexandria offered incomparable learning resources and compatriot intellects to befriend.[6]

Within this environment talent flourished. Archimedes wrote a considerable number of texts on geometry and mechanics, and is considered the father of hydrostatics—the study of the mechanical properties of fluids. Some writings survived; for others we have only references from other sources. How many of his gifts were carelessly lost in the dust of centuries? One expert on the mathematics of antiquity says,

> The treatises are, without exception, monuments of mathematical exposition; the gradual revelation of the plan of attack, the masterly ordering of the propositions, the stern elimination of everything not immediately relevant to the purpose, the finish of the whole, are so impressive in their perfection as to create a feeling akin to awe in the mind of the reader.[7]

Yet he was not content to merely think about problems. Archimedes is justly famous for real-world application of the mechanical principles he wished to understand. Beyond the military defenses for his beloved home city, he also devised more humane applications—creations that extended the normal abilities of humankind. One such invention is the

Seven Archimedes Screws pump wastewater in a treatment plant in Memphis, Tennessee. Each of these screws is 96 inches in diameter and can lift 19,900 gallons per minute. Source: Chris Rorres.

[5] Pappus of Alexandria, *Collection*, Synagoge, Book VIII, c. 340 CE.

[6] There are various references in Archimedes' works to fellow kindred spirits, many of whom studied at the great Library of Alexandria. Likewise, contemporaries also mention Archimedes suggesting a shared background.

[7] Heath, Thomas Little. *Archimedes*, 1897.

so-called Archimedes Screw. It is essentially a tube that follows a helical shape, one end of which is immersed in water. The tube is cranked around its axis, resulting in water being “scooped” into subsequent twists of the helix. It can still be found irrigating the fields of Egypt. But modern instances of this ancient invention turn up in large-scale water control projects, and even in a miniature cardiac pump.

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> Oftimes Archimedes' servants got him against his will to the baths, to wash and anoint him, and yet being there, he would ever be drawing out of the geometrical figures, even in the very embers of the chimney.[8]

You may recall an ancient story: an old man yells “Eureka!” as he leaps from his bath and runs naked through the ancient streets. This was none other than Archimedes, but the caricature does him little justice.

Re-telling the story here shows us the man, clothed or not, who used his mind to fathom more than the bath. The story goes that King Hiero II provided a known weight of pure gold to a craftsman who was commissioned to fashion a crown. Of course the crown to be made must weigh exactly the same as the original gold. When the smith returned the finished crown, it weighed exactly as expected. But the king was suspicious, for metalurgers hid well the knowledge of their arcane craft. Might the smith have mixed the gold with another metal, thus pinching a bit of the royal wealth? Archimedes was summoned to discover if this were so, but commanded not to destroy the crown. In modern terms, he was asked to perform nondestructive analysis.

(Top) The crown (made of an alloy of two metals) exactly balances an equal weight of pure gold.

(Bottom) But if immersed in water the objects will each displace different amounts of the surrounding liquid. Even a slight difference between the objects’ volume will be affected by the buoyant force of the water. Source: Chris Rorres

How would you do it? The problem can seem intractable. And indeed it must have puzzled Archimedes for a time. We can imagine a servant pouring steaming water into a polished cistern, the gentleman stepping in. He relaxes, letting the vapors warm his face and the water sooth his muscles. He forgets his cares. His curious mind wanders. He considers how his body displaces the water. But he asks himself, “What if another body is cast into the water. Suppose a stone that weighs the same as my body is placed here—does it displace the same amount? No. No? No, it does not! The displacement does *not depend on the mass*, but only on the *volume* of the object.” And then connecting this difference to the problem of the crown, Archimedes flashes with insight realizing that two objects made out of *different materials*, but

[8] Plutarch, *Parallel Lives: Marcellus* (C. Rorres)

weighing the same, must always *displace different amounts* of liquid. And if gold were alloyed with another metal it would displace a different amount than an equal weight of pure gold! Stunned to his feet, he cries "Eureka!" (That is, "I have found it!"), and dashes naked from the bath concealing neither modesty nor exuberance from his fellow Syracusans.

Archimedes contrived a device to test the principle—and the crown. The common derivation is that he only had to dunk the crown in a container of water and compare its displacement to that of an equal weight of gold. But this is impractical for very small differences. Instead it is thought that Archimedes used a container in conjunction with a with a balance scale, one or perhaps both arms immersed in the water. Such a device would be consistent with Archimedes' Law of Buoyancy and also the Law of the Lever. As the story goes the crown did displace a different amount than an equal weight of pure gold, and thereby the deceit was discovered—without melting the crown.

History does not record what became of the dishonest—and surely chagrined—goldsmith. But "Eureka!" still echoes from that Syracusan bath 22 centuries ago, reverberating now as our exclamation for discovery. It is a fitting honor to remember who uttered it and why.

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> Among [the Greeks] geometry was held in highest honor; nothing was more glorious than mathematics. But we have limited the usefulness of this art to measuring and calculating.[9]
>
> —Cicero

Despite his achievements in mechanics, Archimedes' great love was geometry. Thales, Pythagoras, Plato, Aristotle, Euclid and others before him mapped out a great new realm of numeral and ratio, logic and proof. Archimedes' love was born within a grand Hellenic flowering of mathematical imagination, even mysticism.

The Greeks did not discover geometry; the Babylonians and Egyptians worked out practical math millennia before the Greeks. The precision of monumental architecture; the complex, if cumbersome numbering systems; the detailed astronomical records, all demonstrate ancient facility with arithmetic and geometry. But the Greeks made something new, something poetic with it. They transformed mathematics into philosophy, into an art form.

The Greeks were passionate about geometry, perhaps the way we are about technology. In our post-modern culture we find much importance from the good, but sometimes troubling things, that technology produces—rapid transportation, global communications, fantastical diversions, biological weapons. So, too, did the Hellenes derive great meaning from the study and application of mathematics. But the Greeks preferred beauty to mere entertainment, sought truth over opinion. Only in this context does a passion like Archimedes' make sense.

[9] Cicero, 106-43 BC, *Tusculan Disputations*, Book I, Section II(5)

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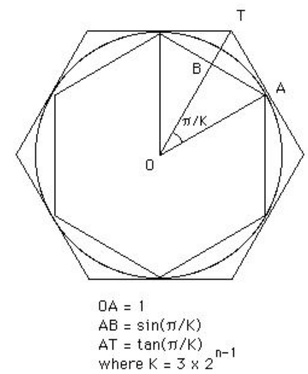
One of the prime examples of this cultural difference is reflected in how the ancients treated the number Pi (π). This curiosity was encountered early in the history of geometry, as it is the ratio of the circumference to the diameter for any circle. But Pi is a tricky number. The more you try to precisely determine the fraction the more Pi resists coming to rest. It keeps hiding somewhere a little beyond the number 3, but can't be pinned down no matter how fine you make the fraction. Pi has, and always will defeat any attempt to squeeze it irrevocably down to some final, finite number. The conservative Egyptians settled on 22/7 as a practical approximation and lived with that for centuries. [10] The Babylonians simply let it go at 3, and tried to not let it bother them. But the Greeks did not let it go. They were obsessed over understanding such things.



Archimedes used an abstract way to estimate Pi. The above diagram shows the sorts of angles and relationships that he used.

Pi and its cousins, the square root of two for example, are strange, haunting things. They are not well-behaved fractions like 1/2 or 2/3. Rather, these perplexing ratios refuse to sit still, and get into all sorts of mischief. The Greeks invented a name for such numbers; they were called *ir*rational or incommensurable. The mystical Pythagoreans, who loved their predictable integers to a fault, were frustrated, embarrassed, and skeptical of these irrationals.

Unlike the Pythagoreans, Archimedes probably found Pi to be more playful than perverse. But he wasn't content to let Pi simply wander off. As a younger man he was determined to tame and adopt Pi.

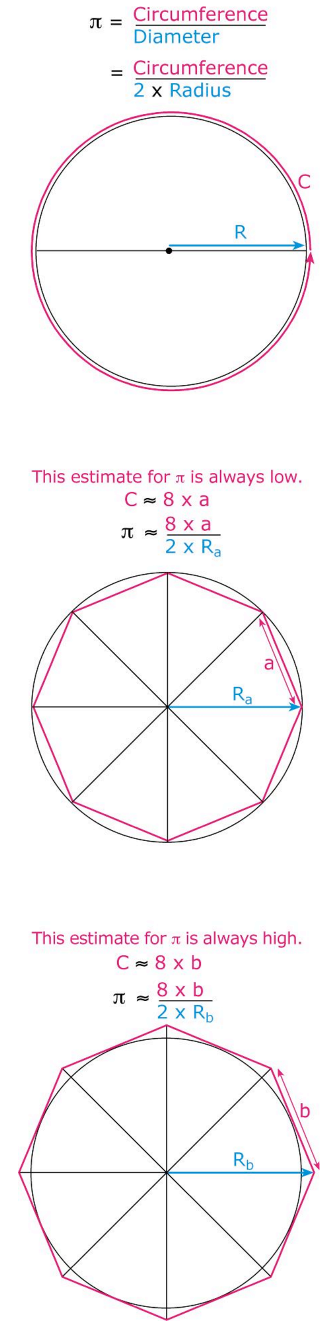
The Archimedian approach involves establishing a polygon boundary both within and just surrounding a circle. Then relying on a certain Euclidian theorem and bisecting ever smaller angles he arrive at limits for Pi. It is a rigorously theoretical, but mind-twisting, tedious calculation.

However it is possible to sense the correctness of his proposal using only basic arithmetic as in the following illustration.[11] First, draw a big circle. From it's center draw a radius line to its circumference. Draw more radii, evenly spacing the resulting slices. It does not matter how many, but for ease of illustration use 8 equal slices. Then connect each radius endpoint (where it meets the circumference) to its nearest neighbors. The result is an 8-sided polygon that crudely approximates the circle. Now simply measure the flat end of any slice and multiply by the number of wedges, in this case 8. This measurement gives an approximation to the circumference. Take that number and divide it by the length of the diameter (remember that the diameter is just twice the

[10] There is conflicting evidence that the Egyptians used a finer result. The Rhind Papyrus, circa 1650 BC, shows Pi at $4(8/9)^2 = 3.16$. However, this seems to be an isolated representation. The more frequent Egyptian records use 22/7 for the value. (Source: C Rorres).

[11] This illustration deviates from the Archimedian method. He did not mechanically add up arbitrary polygon perimeter lengths and divide by the radius. Instead he used geometrical relationships of the angles provided by the polygons so that in principle he wouldn't have to measure anything.

length of the radius). You now have an estimate of Pi. As you can see, the more wedges the closer will be the estimate.



A simple way to estimate Pi.

That is a good place to begin, but realize, as Archimedes did, that such an estimate will always be low: the approximated circumference is smaller than the true one. So repeat the method, now drawing the wedges a little larger, so that the polygon encloses the original circle. Repeat the calculation. This, too, will provide an estimate of Pi, this new one always being a little larger.

Archimedes did this using 96 wedges and found that Pi was a smidgen more than 3 10/71, but a shade less than 3 1/7. Averaged and converted to decimal notation this is 3.1418, an error of only about 2 in the ten thousandth's place. As the number of polygon sides increase the limits begin to converge, pressing out, as between two rollers, a finer and finer result for Pi. This method is considered the first theoretical approximation for Pi and foreshadows the infinitesimals that Newton would use to place the Earth and moon in their orbits.

We must remember that Archimedes did not have algebraic or decimal notations to help him; it was the equivalent of using manual labor to build pyramids. Commentators note that it was a "stupendous feat both of imagination and of calculation and the wonder is not that he stopped with polygons of 96 sides, but that he went so far." [12]

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Throughout his career Archimedes loves reasoning things out. The Roman author Plutarch writes, "It is not possible to find in all geometry more difficult and intricate questions, or more simple and lucid explanations."

In one of his earliest works Archimedes shows his proof that a sphere embedded in a cylinder just tight enough to enclose it would have a ratio of 2 to 3. That is, the volume of the sphere would be exactly 2/3 the volume of the smallest cylinder to contain it. (He also finds that the surface areas of both objects will also have the same relationship.) It is this image and this proof that a young Archimedes is justly proud; in later years he will ask that his grave be marked with this accomplishment. To modern ears this might sound trivial, odd, maybe irrational. But to the Greek mind nothing was so fine as to apprehend nature through reason.

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[12] J J O'Connor and E F Robertson, *Pi through the ages*, web document, 2001.

In *The Sandreckoner*, perhaps his last work, Archimedes seeks to find the volume of the universe itself—audacity itself when one learns the proposal is to find how many grains of sand will do the job. Working from available astronomical conjectures, Archimedes considers how he might express such a vast sum. His incredible assumption: *the universe is not infinite*. He's proposing something actually measurable, at least in principle. With our modern exponential notation we take a problem like this for granted. But the notations of his day were cumbersome beasts of burden, difficult to master and not made to cover great distances. So Archimedes proposes his own alternative notation, which can express a number equivalent to $8x10^{16}$. That's how many grains of sand he reckons will fill the universe.

Archimedes must suspect that his answer is at best a guess, but that is not the point. What he wants to learn is *how* to learn. What tools, what methods, what reasoning will work? And he desires such tools, especially conceptual ones, because they reveal, at least a little, the incommensurable beauties in the universe.

It is a credit to his mind that the tool he chooses in *The Sandreckoner*, the thing he uses to scale the very large, will be from the very small—no more than a grain of sand. What better example of humility and courage can Reason offer? Like the artist or the mystic, Archimedes exalts in contrasts of the immense. But unlike them, and more like the philosopher, Archimedes seeks the *measure* of eternity, a humble yet determined eagerness to comprehend the universe, if only with imagination.

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But, if you'll recall, the Romans are camped outside the gates of Syracuse. Archimedes' war machines frustrate Marcellus and his legions for eight months. But then, during a festival the Syracusans celebrate to indulgence. The Romans capitalize on their enemy's negligence, breach the gates by stealth, and thereby capture the city. Marcellus gives his victorious soldiers a single day to loot and plunder, as was their custom. But Marcellus also issues orders to bring Archimedes before him, no doubt to meet, and perhaps congratulate, the nemesis of Roman power.

Meanwhile, unaware or unconcerned with events, Archimedes is lost in thought, concentrating on a geometrical theorem. As the Romans round up the citizenry a soldier comes upon the old Greek drawing diagrams on the floor, as was his fashion. The impatient soldier commands the septuagenarian to leave. But the white haired geometer waves him away; what could possibly be more important than this peaceful reverie with beauty? Irritated at being dismissed, and perhaps unaware of the value of his find, the soldier draws his sword and strikes down Archimedes for the sake of convenience.[13] Irony and tragedy alloy in this moment, the baser motive corrupting the nobler spirit. Alfred North Whitehead, no mean mathematician himself, described the imbalance this way:

[13] Plutarch briefly recounts three different versions of the death of Archimedes, while Livy offers another. All versions agree that Archimedes was killed at the hands of a Roman soldier, and most suggest that Archimedes was concentrating on some geometrical problem.

Plutarch, c.45–119 CE, *Parallel Lives: Marcellus*.

Livy, 50 BC –17 CE, *History of Rome from its Foundation*, Book XXV

> The death of Archimedes by the hands of a Roman soldier is symbolical of a world-change of the first magnitude… The Romans were a great race, but they… were not dreamers enough to arrive at new points of view… No Roman lost his life because he was absorbed in the contemplation of a mathematical diagram.[14]

In the wars with Carthage, Rome takes Syracuse and all of Sicily—then in more wars, all the lands that border the Mediterranean, and more beyond, and thus becomes a great empire. It adopts some, but by no means all, of what Hellenic civilization offers. The Romans hire Greeks to teach them, and some things the Romans learn. The victors build stadiums, aqueducts and roads with the fruits of geometry. But did the Romans, and do we, fail to appreciate the better wealth in such gifts?

137 years after the death of Archimedes, the Roman orator Cicero searches Syracuse for the tomb.[15] After some hunting he finds the site, the current inhabitants evidently unaware of it. Weeds and overgrowth obscure the grave. On a small pillar Cicero discovers an inscribed image—a sphere within a cylinder and a few words of proof—Archimedes testament to his first love and to the gifts he left for us. These gifts are for the ages, although perhaps too archaic for modern tastes. After all, physics has rocketed far beyond his laws of the lever and buoyancy. The esoteric geometrical proofs seem quaint and dusty next to our slick wave functions and non-Euclidian geometries. But if they do, perhaps like the Romans, we flatter ourselves to think we've done so well. Archimedes had less to work with, yet managed to show that reason and imagination, harnessed together, give us the means to comprehend beauty. It is a beauty he relished to his dying moment. Would that we could live so well.

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[14] Alfred North Whitehead, *An Introduction to Mathematics*, p 25-26, 1958 ed.

[15] Cicero, 106 – 43 BC, *Tusculan Disputations*, Book V.